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Scaling law in electrospinning: relationship between electric current and solution flow rate

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Abstract

A modified equation for charge conservation in electrospinning is suggested, and a nonlinear relation between the electric current of charged jet and the solution flow rate is obtained.

Keywords: Electrospinning; Polymer; Allometric scaling

1. Introduction

Electrospinning [1–10] is an economical and simple method used in the preparation of polymer fibers. The fibers prepared via this method typically have diameters much smaller than is possible to attain using standard mechanical fiber-spinning technologies. This technology is increasingly becoming very popular in the preparation of polymer fibers either in the form of individual fibers or non-woven fiber mats.

Electrospun polymer fibers with diameters in the range from several micrometers down to tens of nanometers are of considerable interest for various kinds of applications. It is now possible to produce a low cost, high-value, high-strength fiber from a biodegradable and renewable waste product for various kinds of applications. The porous structured electrospun membrane as wound dressing [11] can exudates fluid from the wound, does not build up under the covering, and does not cause wound desiccation. The electrospun nanofibrous membrane shows controlled evaporate water loss, excellent oxygen permeability, and promoted fluid drainage ability, but still it can inhibit exogenous microorganism invasion because its pores are ultra-fine. Other examples are thin fibers for filtration application [12,13], bone tissue engineering [14], drug delivery [15], catalyst supports [16], fiber mats serving as reinforcing component in composite systems [17], fiber templates for the preparation of functional nanotubes [18].

Many experiment data shows a power law relationship between electric current (I) and solution flow rate (Q) under the condition of fixed voltage [6,19]:

$$I \sim Q^{\alpha}, \tag{1}$$

where α is the scaling exponent. When $\alpha = 1$ the relationship is isometric, and it is allometric [20–23] when $\alpha \neq 1$. The value of the exponent α is dependent upon the solution. Theron et al. obtained the mean values of α for different solutions of PEO, PAA, PVA, PU and PCL [6]. Ref. [19] reports that in the case of electrospraying of 1-octanol seeded with sulfuric acid there also exists a power law relationship between I and Q with an exponent value of 0.5.

2. Allometric scaling relationship between current and solution flow rate

The charged jet can be considered as a one-dimensional

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flow. Conservation of mass gives

$$\pi r^2 u\rho = Q, \tag{2}$$

where Q is the mass flow rate, r is radius of the jet, u is the velocity, ρ is density.

The equation for current balance reads [4]

$$2\pi r u\sigma + k\pi r^{\frac{2(\delta+1)}{2\delta+1}} E = I, \tag{3}$$

where σ is the surface charge, E is the intensity of the applied electric field, and I is the current passing through the jet, δ is a constant. When $\delta=0$, Eq. (3) turns out to be the classical equation for charge conservation. The value of parameter δ depends upon conductivity and polymer concentration.

Force balance gives

$$\frac{d}{dz}\left(\frac{u^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{2\sigma E}{\rho r} + \frac{1}{r^2}\frac{\partial \tau}{\partial z}, \tag{4}$$

where p is the internal pressure of the fluid, ρ is the liquid density, τ is viscous force.

Hereby we will apply an allometric approach to the establishment of a scaling relation between I and Q. Allometric approach is in common use in biology [20–24], but is significantly less familiar in polymer science, although allometry involves few new ideas, it leads to vast simpler, more transparent expressions for the interpretation of many complex phenomena in electrospinning than those provided by traditional ways in open literature. In our previous paper [4], we establish an allometric scaling relation between electric current and voltage, in this paper we extend our work to the scaling relation between I and Q.

Assume that the intensity of the applied voltage (E) keeps unchanged during the electrospinning procedure, i.e. $E\sim r^0$, and assume that

$$Q \sim r^a, \tag{5}$$

where a is a non-zero scaling exponent.

This assumption concerns processes with mass exchange between the jet and a gaseous surroundings. In case of zero mass exchange, i.e. without the evaporation, we have $Q=$ constant, the scaling exponent becomes zero, such phenomenon was studied in our previous publication [4].

From (2) and (3) we have

$$I \sim r^{\frac{2(\delta+1)}{2\delta+1}}, \tag{6}$$

$$u \sim r^{a-2}, \tag{7}$$

and

$$\sigma \sim r^{\frac{4\delta+3}{2\delta+1}-a}. \tag{8}$$

Combining (5) and (6), (5) and (8), we obtain

$$I \sim Q^{\frac{2(\delta+1)}{a(2\delta+1)}}, \tag{9}$$

and

$$I \sim \sigma^{\frac{2(\delta+1)}{(4-2a)\delta+3-a}}. \tag{10}$$

Under the condition of fixed voltage, the relation of I and Q can be expressed as

$$I = \beta Q^{\frac{2(\delta+1)}{a(2\delta+1)}}, \tag{11}$$

where β is a constant. The three parameters (β, a and δ) in Eq. (11) can be determined experimentally. Measure the current when solution flow rate changes:

$$I_1 = \beta Q_1^{\frac{2(\delta+1)}{a(2\delta+1)}}, \tag{12}$$

$$I_2 = \beta Q_2^{\frac{2(\delta+1)}{a(2\delta+1)}}, \tag{13}$$

$$I_3 = \beta Q_3^{\frac{2(\delta+1)}{a(2\delta+1)}}. \tag{14}$$

From (12)–(14), the three parameters(β, a and δ) can be easily identified.

3. Conclusion

We obtain a scaling relation between electric current and solution flow rate, which is able to describe a complex dynamic process from the theory, and it requires less empirical or semi-empirical input. Of course the authors understand that no matter how rigorous, some experimentally verification is needed to validate the model. A thorough such experimental work is under way and the results will be reported in future.

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